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MATHEMATICAL METHODS
Teach Yourself Series
Topic 5: Transformations of Functions

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Contents

Transformations of Functions

Type 1 – Functions

As it appears in Unit 1

All graphs can be transformed by the following:

Reflections: $-f(x)$

Reflection across *x* axis. *y* coordinates on the original functions are multiplied by -1.

$f(-x)$

Reflection across *y* axis. *x* coordinates on the original functions are multiplied by -1.

Dilations $af(x)$

Dilation of *a* from *x* axis (parallel to *y* axis) – *y* coordinates on the original functions are multiplied by "*a*".

$f(ax)$

Dilation of $\frac{1}{a}$ from *y* axis (parallel to *x* axis) – *x* coordinates on the original functions are multiplied by "$\frac{1}{a}$".

Translations $f(x)+k$

Translation of *k* along *y* axis (from *x* axis) – *y* coordinates on the original functions have "*k*" added to them.

$f(x-h)$

Translation of *h* along *x* axis (from *y* axis) - *x* coordinates on the original functions have "*h*" added to them.

Watch for the tricky questions – The dilation from *y*
The transformations given out of order

Example

Describe the transformations that have been applied to $\boldsymbol{f(x) = \sqrt{x}}$ to result in the function $\boldsymbol{f(x) = -2\sqrt{3(x-1)} + 4}$.

Answer:

Reflection across the *x* axis.

Dilation of 2 units from the *x* axis and $\frac{1}{3}$ of a unit from the *y* axis.

Translation of 1 units along *x* axis and 4 units along *y* axis.

Calculator skills

Define functions
Type transformation in function form once function is defined.

Type 2 – Matrices

As it appears in Unit 3

Reflection in *x* axis $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Reflection in *y* axis $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Reflection across $y = x$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Dilation from *x* axis $\begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$

Dilation from *y* axis $\begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$

Translation along *y* axis $\begin{bmatrix} 0 \\ k \end{bmatrix}$

Translation along *x* axis $\begin{bmatrix} h \\ 0 \end{bmatrix}$

Note: You put the transformations in to the matrices as you read them.

- Have to solve the matrix equation: $X = TX' + B$

  $X = \begin{bmatrix} x \\ y \end{bmatrix}$

  $T = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ ie dilation and reflection matrix

  $X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

  $B = \begin{bmatrix} h \\ k \end{bmatrix}$ ie translation matrix

- Solve for X’
- Swap *x* for *x’* and *y* for *y’* in the rule that is being transformed

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Solutions to Review Questions

1.

$$f(x) = 3f(-(x-2))$$

$$= \frac{3}{-(x-2)}$$

$$= \frac{3}{2-x}$$

2.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3x + 2 \\ -2y + 7 \end{bmatrix}$$

$x = \frac{x' - 2}{-3}$ and $y = \frac{y' - 7}{-2}$ Substitute these into the function to be transformed and transpose to the answer.

$$f(x) = 2\sqrt{-\frac{1}{3}(x-2)} + 7$$

3.

$$-2f\left(\frac{x}{3}\right) + 3 = -\frac{2}{\left(\frac{x}{3}\right)^2} + 3$$

$$f(x) = -\frac{18}{x^2} + 3$$

4.

Transpose function to $\frac{2}{-(x-2)} + 4$ before describing transformations.

Reflection in *y* axis

Dilation of 2 units from *x* axis

Translation of 2 units along *x* and 4 units along *y*.

5. Apply transformations in order to get:

$$-2f\left(\frac{x}{3}\right) + 3 = -2e^{\frac{x}{3}} + 3$$

6.

Set up the following matrix equation: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$x' = 4x$ $\quad y' = -\frac{y}{2} + 3$

$x = \frac{x'}{4}$ $\quad y = -2(y' - 3)$

Make these substitutions and transpose to make *y'* the subject

$y = -\frac{1}{128}x^3 + 3$

7.

Apply the transformations $-5f(2x)+3$

$y = -5\log_e(2x) + 3$